Basic circuits and laws

（電路學補充資料（四電機二A））

Basic DC circuits and laws are the basis of three-phase AC systems. Students will benefit from this knowledge when they start analyzing an AC system which is the goal of this course.

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networks and circuits

- The interconnection of two or more simple circuit elements forms an electrical network.
- If the network contains at least one closed path, it is also an electric circuit.
- A point at which two or more elements have a common connection is called a node.
- If we start from one node and end at another node without encountering any node more than once, then what we have passed through is a path.
- A closed path is called a loop.
- A branch is a single path in a network composed of one simple element and the node at each end of that element.

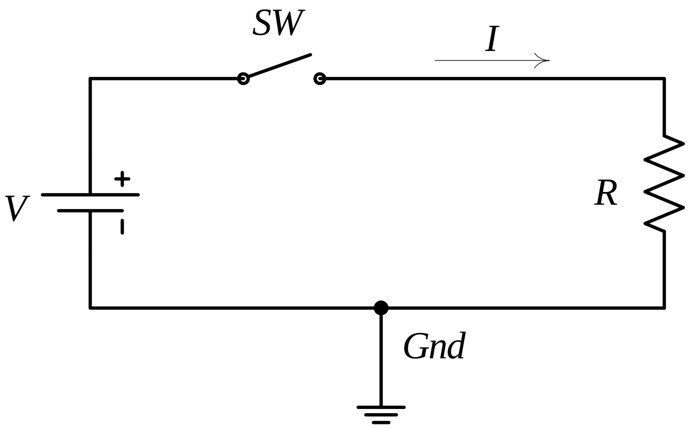
Circuit components

- Electrical Energy Sources: voltage or current sources, parallel and/or series
- Conductors: wire, cable
- Insulators: wire or cable skin
- Load: light bulb (lamp), heater
- Control device: a switch
- Protection device: fuse or breaker

From physical devices to symbols

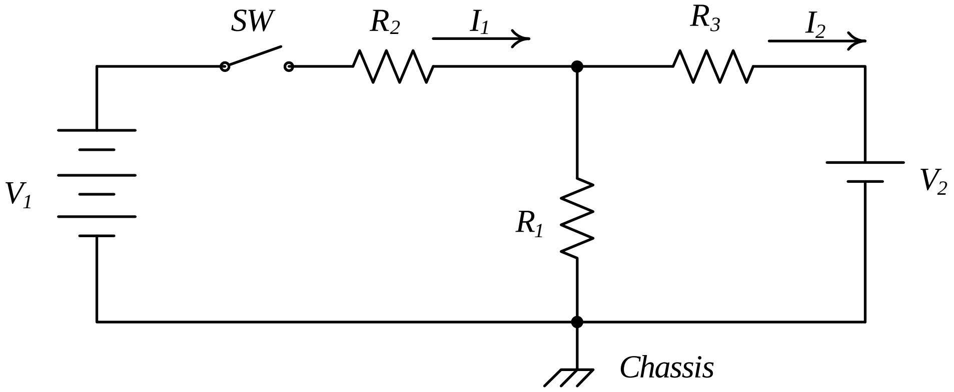
- Circuit symbols
- Circuit diagram
- Common ground
- Polarity
- Conventions
- Sources: voltage or current
- Parallel and series

A basic circuit



Gnd means grounding point which is a voltage reference point where the voltage is normally designated to be zero volt.

A slightly more complicated circuit



Chassis is also often taken as a voltage reference point where the voltage is zero volt.

Subscripts “1”, “2” and “3” are used to identify different resistances or voltages or currents in a circuit diagram.

Ohm’s law

- Equation: *V=IR or I=V/R or I=GV*
- The current-voltage relation of a (linear) resistance

$$I = \frac{1}{R}V \qquad i = \frac{1}{R}\cdot v \qquad i(t) = \frac{1}{R}\cdot v(t)$$

- In DC analysis, average value is also instantaneous value. It is not the case in AC analysis.

Power calculation

- *P=VI why?*

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta W}{\Delta q}\cdot\frac{\Delta q}{\Delta t} = VI$$

- By applying Ohm’s law, power can be expressed in several ways:

$$P = VI = I^2R = \frac{V^2}{R}$$

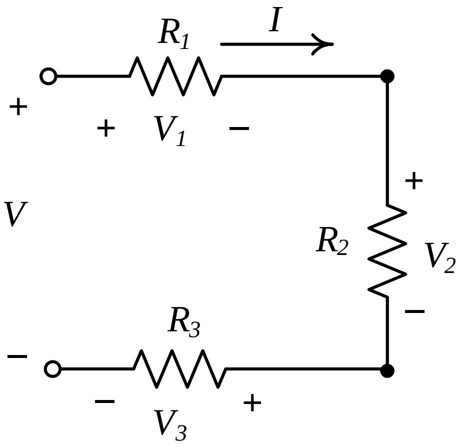
Joule's law – basic principle of electrical heating

- This law states the mathematical description of the rate at which resistance in a circuit converts electrical energy into heat energy. The English physicist J. P. Joule discovered in 1840 that the amount of heat per second that develops in a wire carrying a current is proportional to the electrical resistance of the wire and the square of the current.

$$Q = I^2 R \cdot t$$

$$\text{heat energy} = (\text{electric power}) \cdot t$$

Series circuit



In a series circuit, all the components have the same current.

$$I_{R1} = I_{R2} = I_{R3} = I$$

$$V = V_{R1} + V_{R2} + V_{R3} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

Equivalent resistance $R_T = R_1 + R_2 + R_3$

Series circuit

- Kirchhoff’s voltage law (KVL)

$$V = V_{R1} + V_{R2} + V_{R3}$$

- The sum of the voltage drops around a circuit equals the applied voltage.
- Power (energy) conservation

$$P_T = P_{R1} + P_{R2} + P_{R3}$$

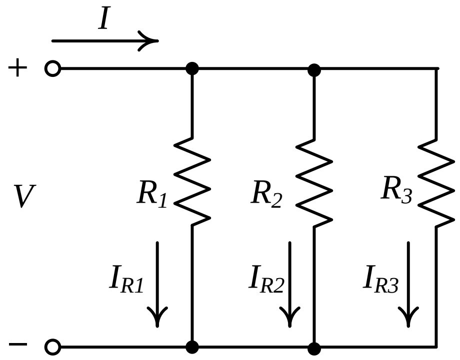
Series circuit

Voltage-divider（分壓器）equation

$$V_{Rk} = V \cdot \frac{R_k}{R_T}$$

R_k is any one of the resistances in the series circuit.
R_T is the sum of all the resistances.

Parallel circuit



In a parallel circuit, all the components have the same voltage.

$$V_{R1} = V_{R2} = V_{R3} = V$$

$$I = I_{R1} + I_{R2} + I_{R3} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Equivalent resistance $R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

Parallel circuit

- Two parallel resistances R_1 and R_2 have the equivalent resistance R_T

$$R_T = R_1 \,||\, R_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 \times R_2}{R_1 + R_2}$$

- This formula is often used, and is easily memorized as “product over sum” of two resistances.
- Note that this formula applies strictly to the case of two resistances in parallel only.

Parallel circuit

- Kirchhoff's current law (KCL)

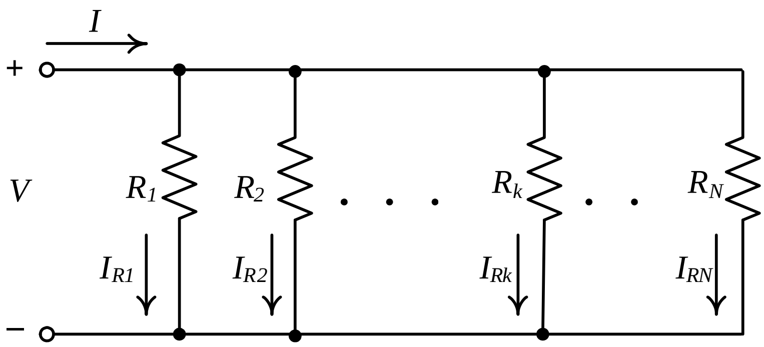
$$I = I_{R1} + I_{R2} + I_{R3}$$

- The sum of the currents entering a junction (node) equals the sum of the currents leaving a junction (node).
- Power (energy) conservation also applies in parallel circuits.

$$P_T = P_{R1} + P_{R2} + P_{R3}$$

Parallel circuit

- Current-divider（分流器）formula



$$I_{Rk} = \frac{V}{R_k} = \frac{I \cdot \left(1/R_1 + 1/R_2 + \cdots 1/R_N\right)^{-1}}{R_k} = \frac{I}{G_T R_k}$$

G_T is the total conductance, $G_T = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$

Current-divider formula

- The current-divider formula states that the current passing through a parallel branch is inversely proportional to the branch's resistance.

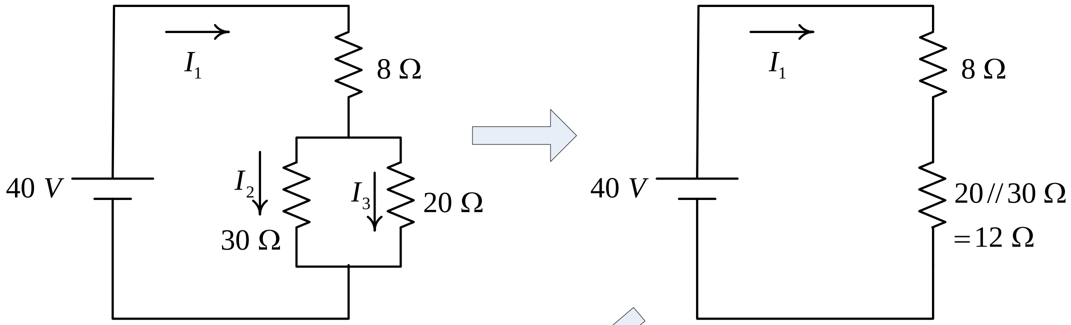
$$I_{Rk} = \frac{I}{G_T R_k}$$

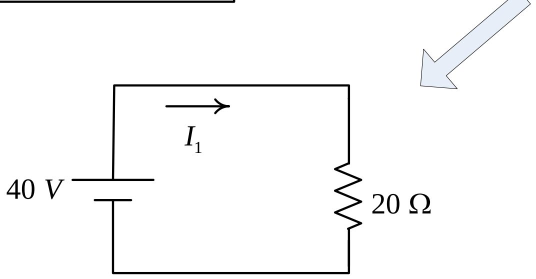
- Current always finds a path with least resistance to flow. I am used to comparing an electric current with students taking courses, for students also look for a course with least *resistance*.

Series-parallel circuits

- Some of the features of the series circuit and the parallel circuit are incorporated into series-parallel circuits.

Circuit simplification using parallel and series combination - example



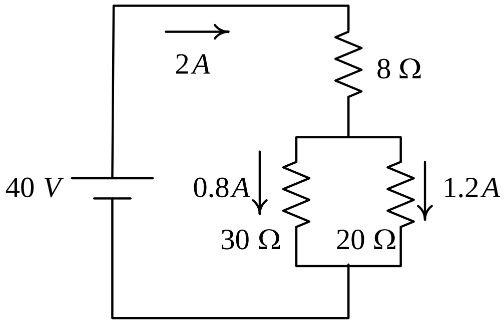


$$I_1 = \frac{40}{20} = 2A$$

$$I_2 = 2 \times \frac{20}{20+30} = 0.8A$$

$$I_3 = 2 - 0.8 = 1.2A$$

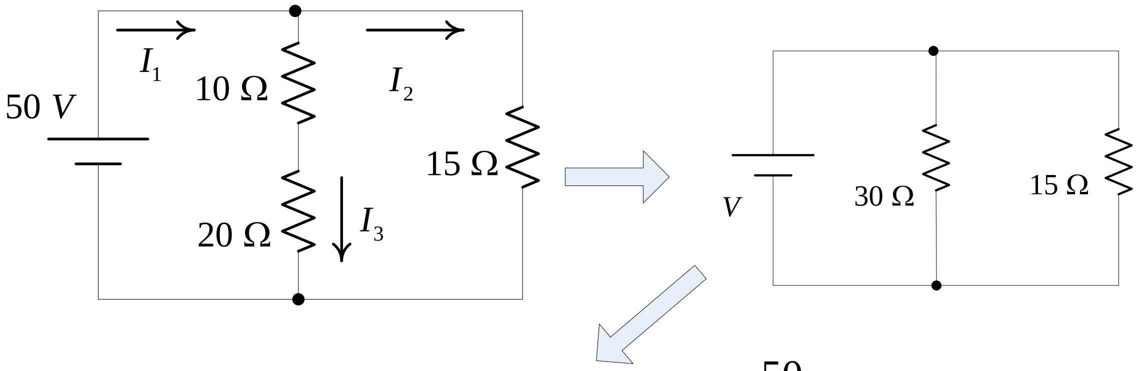
Energy balance analysis: calculate the power supplied by voltage source and the power dissipated by resistances.

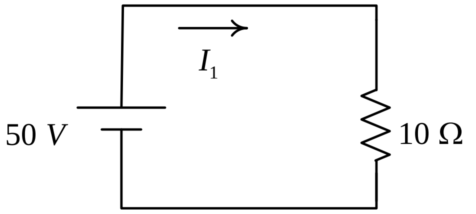


$$\left.\begin{aligned} P_{R1} &= I_1^2 R_1 = 2^2 \times 8 = 32W \\ P_{R2} &= I_2^2 R_2 = 0.8^2 \times 30 = 19.2W \\ P_{R3} &= I_3^2 R_3 = 1.2^2 \times 20 = 28.8W \end{aligned}\right\} total\ 80W$$

$$P_{source} = VI_1 = 40 \times 2 = 80W$$

Circuit simplification using parallel and series combination – one more example





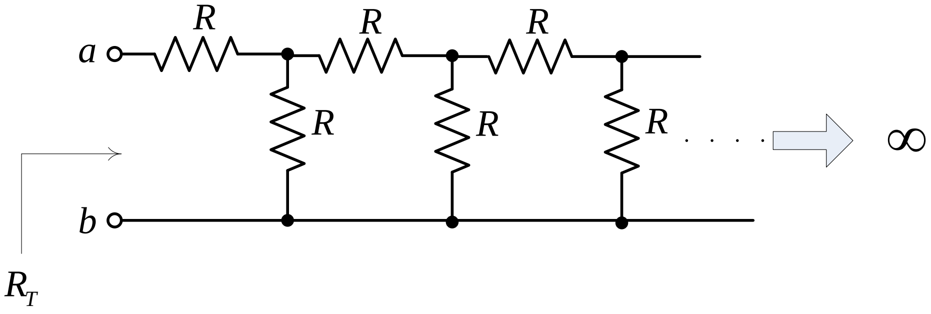
$$I_1 = \frac{50}{10} = 5A$$

$$I_2 = 5 \times \frac{30}{15+30} = 3.33A$$

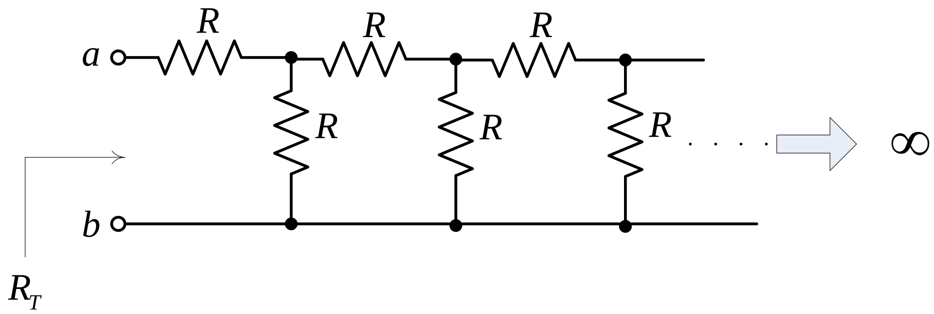
$$I_3 = 5 - 3.33 = 1.67A$$

Some interesting questions about series-parallel resistance circuit

- A ladder circuit is composed of an infinite number of identical resistances of R-Ω as shown below. What is the resistance R_T looking into terminals a and b?



Golden ratio



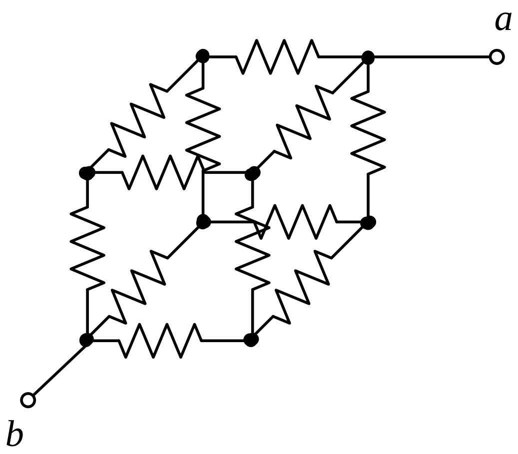
Golden ratio is implied in this ladder circuit. Can you find it? It is interesting to know the golden ratio hidden in electric circuit quantities.

What is golden ratio? For more information, please refer to the Wikipedia

http://en.wikipedia.org/wiki/Golden_ratio

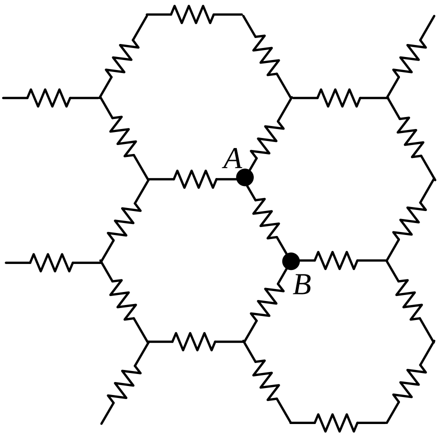
Resistor cube

- We use 12 resistors of 1 Ohm to construct a cube as shown below. Find the resistance between terminals *a* and *b*?



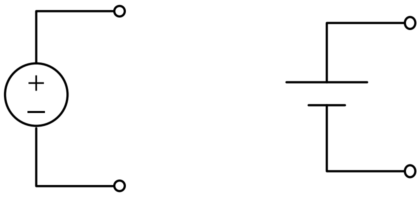
Honeycomb circuit configuration

- An infinite number of identical 1 Ω resistors are connected to form a honeycomb circuit, as shown in the figure to the right. What is the resistance between A and B?

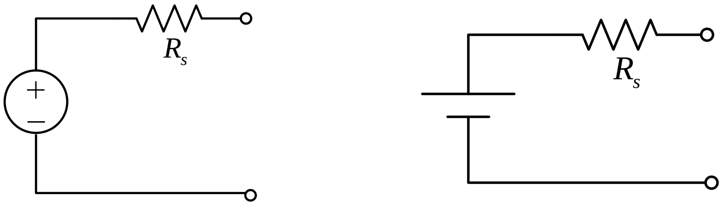


Electrical sources

- Ideal voltage source

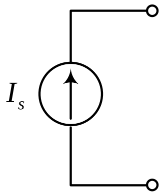


- Practical voltage source (with internal resistance)

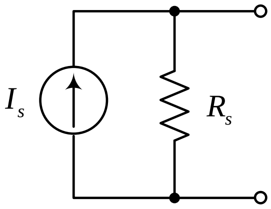


Electrical sources

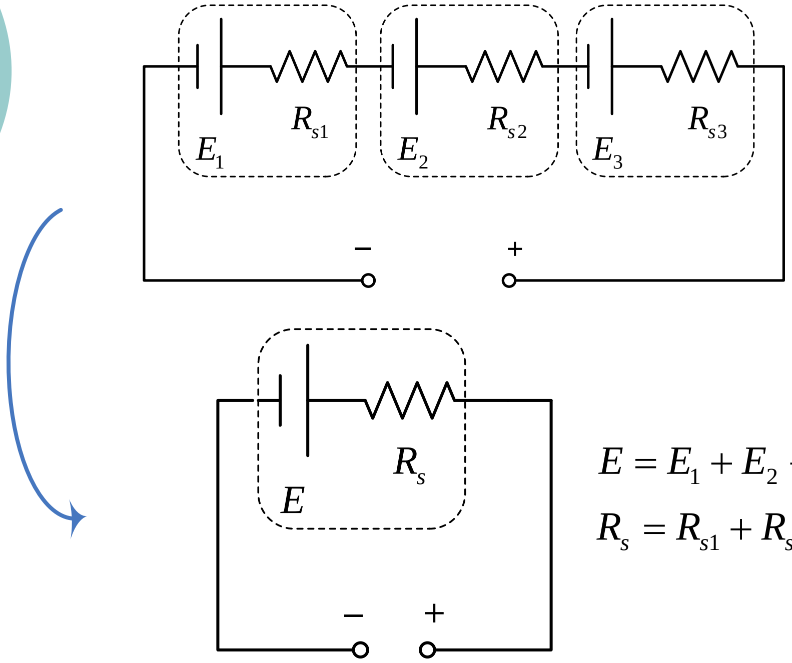
- Ideal current source



- Practical current source (with internal resistance)



Batteries in series

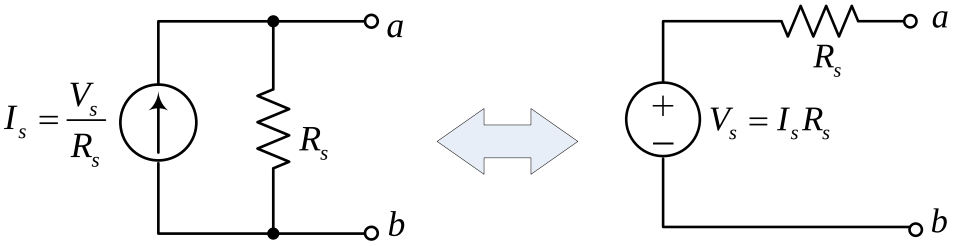


$$E = E_1 + E_2 + E_3$$

$$R_s = R_{s1} + R_{s2} + R_{s3}$$

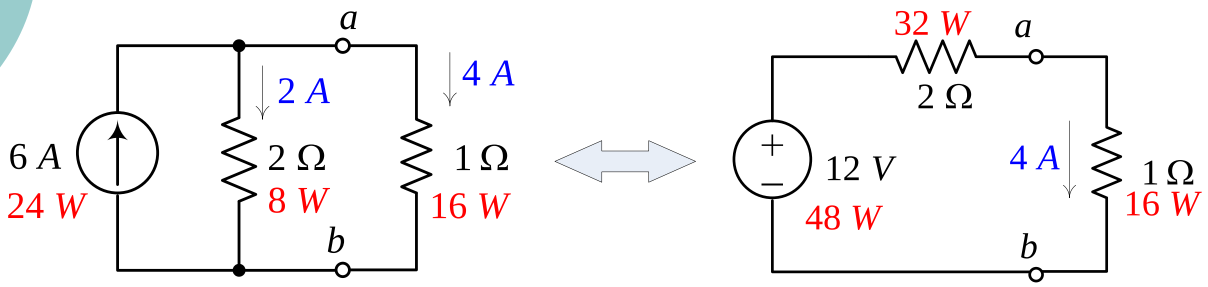
Equivalent sources

- A voltage source can be replaced by an equivalent current source, and vice versa.



Note: The equivalence holds only for the circuit connected to the source terminals *a* and *b*, not for the source itself.

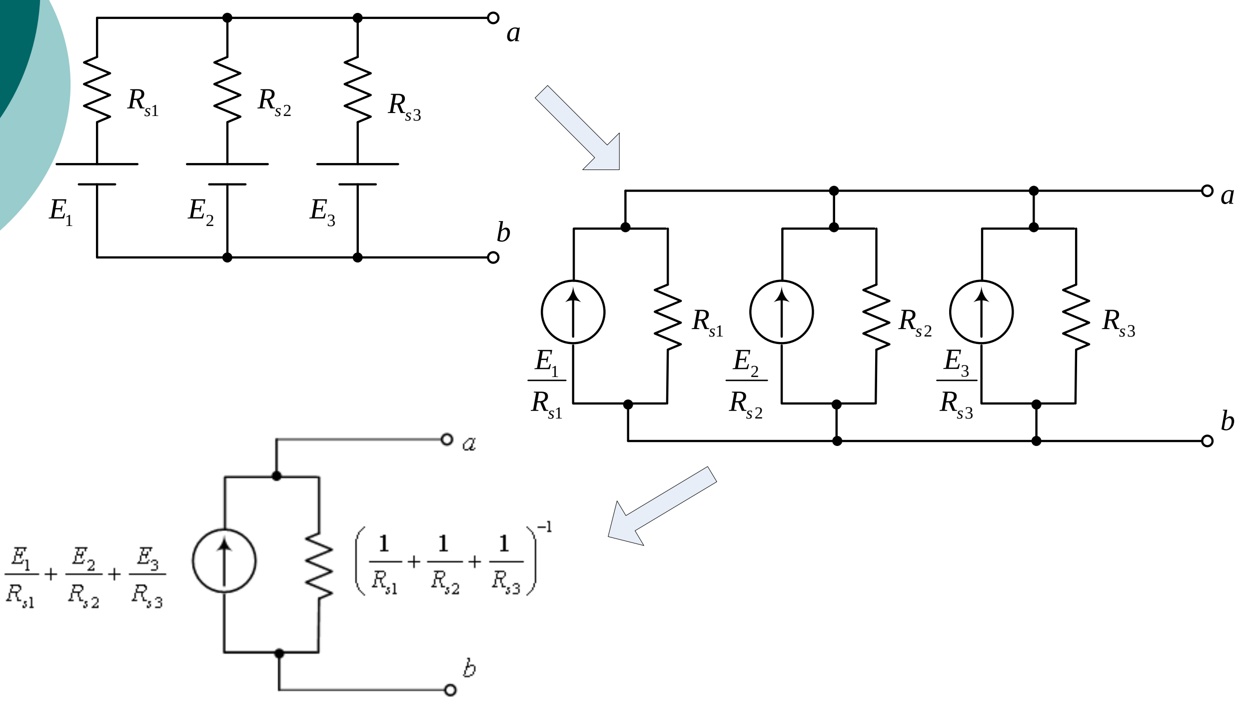
Source equivalence and non-equivlence



The 1-Ohm loads in the two circuits have exactly the same electrical quantities (voltage, current and power), but to the left of terminals a and b, the electrical quantities of the two sources and two internal resistances are NOT the same.

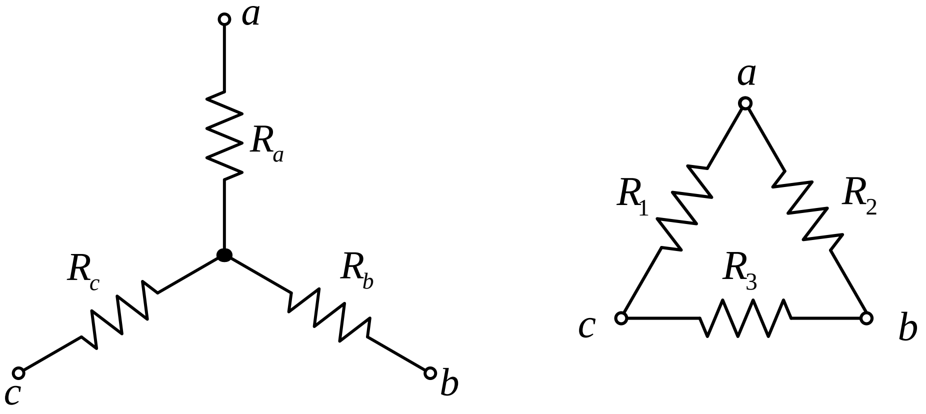
Parallel batteries: Source simplification

Millman’s theorem （密爾曼定理，大部份的電路學教科書都未提及該定理，很多人都不知道Millman是何許人也？）



More complicated circuits
Y and Δ networks

- ○ Y- (star) and Δ- (triangle or delta) networks



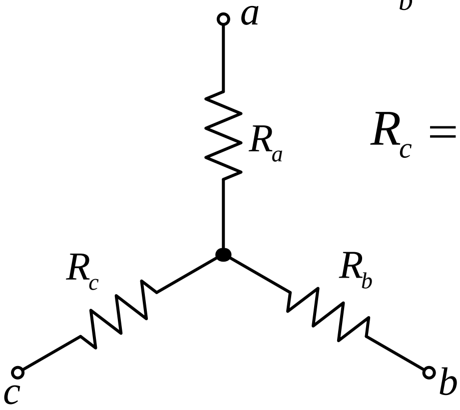
The names used to identify the individual terminals and resistors of the two networks should be memorized.

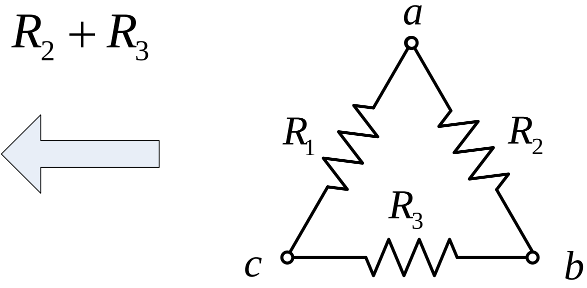
Conversion from Δ to Y

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$



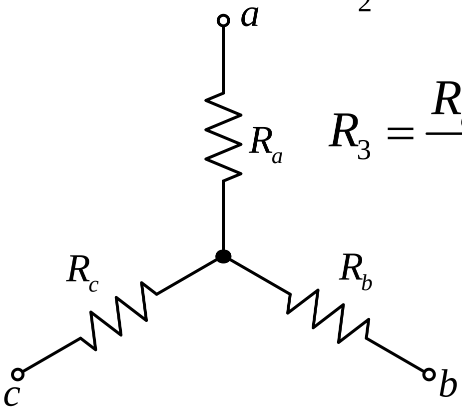


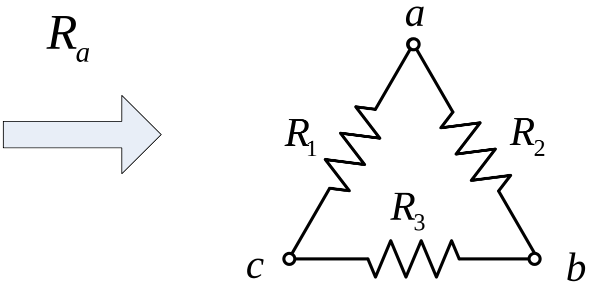
Conversion from Y to Δ

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$





Application of Δ-Y and Y-Δ conversions

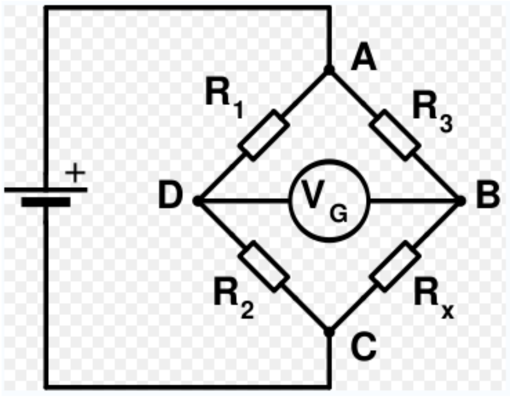
- There are many circuits which cannot be solved by techniques applicable to series, parallel or series-parallel circuits. However, it is usually possible to solve these circuits using Δ-Y and/or Y-Δ conversions.
- An example: *Wheastone bridge* （該聽說過「惠斯登電橋」吧！）

Charles Wheatstone

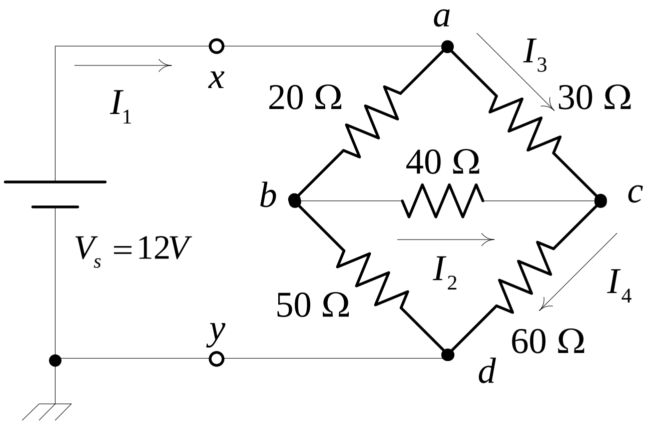
- Sir Charles Wheatstone FRS (6 February 1802 - 19 October 1875), was a British scientist and inventor of many scientific breakthroughs of the Victorian era, including the English concertina, the stereoscope (a device for displaying three-dimensional images), and the Playfair cipher (an encryption technique). However, Wheatstone is best known for his contributions in the development of the Wheatstone bridge, originally invented by Samuel Hunter Christie, which is used to measure an unknown electrical resistance, and as a major figure in the development of telegraphy.

Wheastone bridge: measurement of resistance

$$R_x = (R_2/R_1) \cdot R_3$$



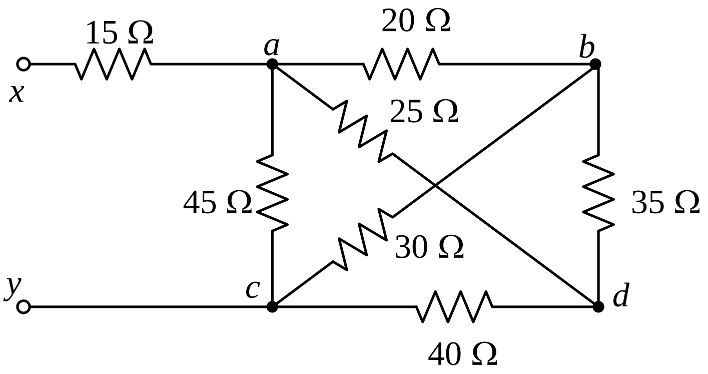
Wheastone bridge circuit



For the above Wheastone bridge circuit, please find currents I_1, I_2, I_3, and I_4 ,and voltages V_b and V_c.

Possible solutions: (0.305A, 0.00775A, 0.1282A, 0.13595A, 8.464V, 8.154V)

Another example of a more complicated circuit

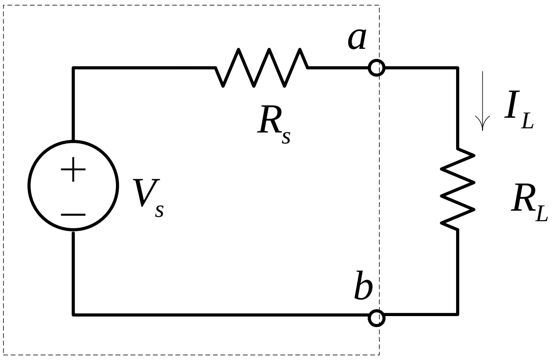


Find the resistance between terminals x and y of the circuit above. (32.36Ω)

Maximum power transfer theorem

- A practical DC source can be modeled by an ideal voltage source V_s in series with an internal resistance R_s .
- The load resistance R_L may vary from 0 Ω (short-circuit) to infinity (open-circuit).
- The maximum *fault current* occurs when R_L is short-circuited.

$$I_f = V_s/R_s$$



Maximum power transfer theorem

- Load current and the power delivered to the load are

$$I_L = \frac{V_s}{R_s + R_L} \qquad P = I_L^2 R_L = \frac{V_s^2 R_L}{\left(R_s + R_L\right)^2}$$

- The load power depends on the value of R_L. The maximum power can be found by the derivative

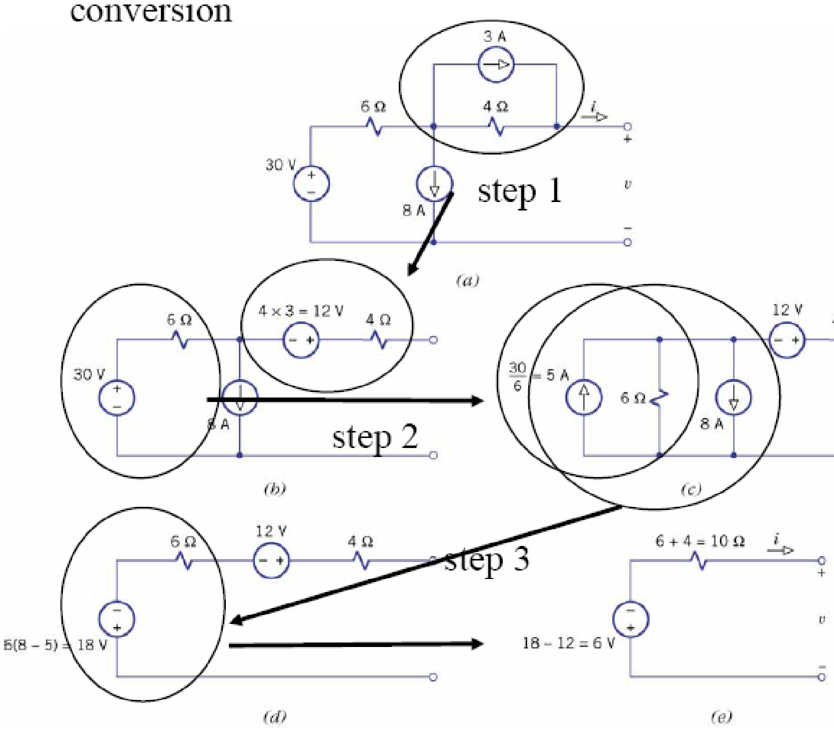
$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V_s^2 R_L}{\left(R_s + R_L\right)^2} = 0 \Rightarrow R_L = R_s$$

- Maximum power transfer theorem (MPTT): *Maximum power is transferred to a load when the load resistance is equal to the internal resistance of the source.*

Maximum power transfer theorem

- When maximum power transfer occurs, the efficiency is 50% (why?).
- Applications of the MPTT: load matching: telephone line, electronic signals, amplifier design…etc.
- Question: What is the value of R_L for the *maximum efficiency* transfer?

12. Ex. 2.16 (ex.2.10) find Thevenin equivalent circuit using source conversion



$R_L = 2$

$$\rightarrow i = \frac{-6}{10+2} = -0.5A$$

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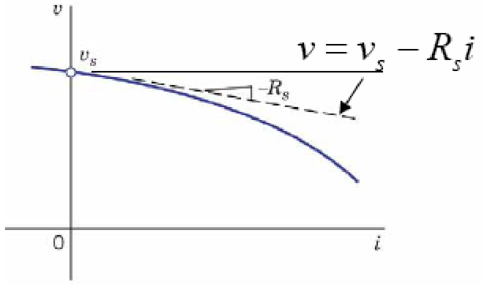
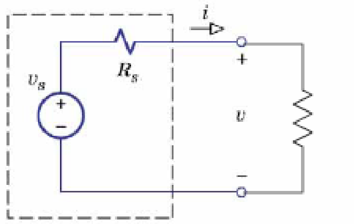
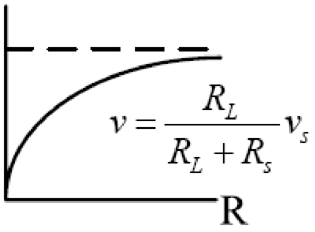
1. real source models

“good” voltage source: $v \sim v_s$, $R_s << R_L$



$v = v_s - R_s i$



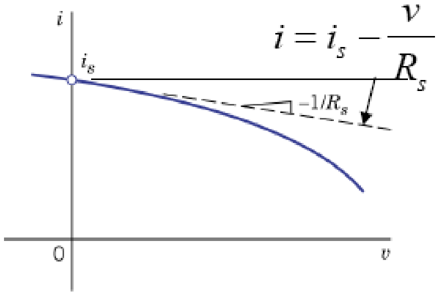


$$v = \frac{R_L}{R_L + R_s} v_s$$

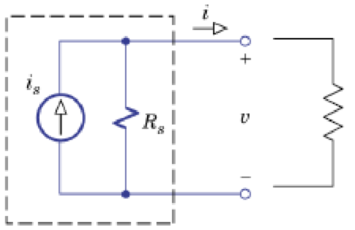
(a) v-i curve of a real voltage source

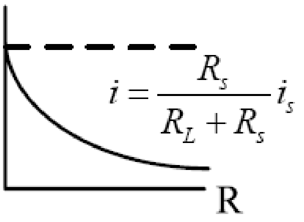
(b) Lumped-element model for $|i| \ll v_s/R_s$

“good” current source: $i \sim i_s$, $R_s >> R_L$



$$i = i_s - \frac{v}{R_s}$$





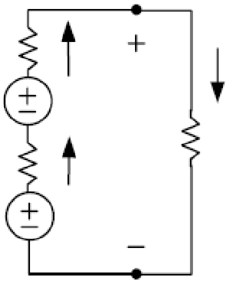
$$i = \frac{R_s}{R_L + R_s} i_s$$

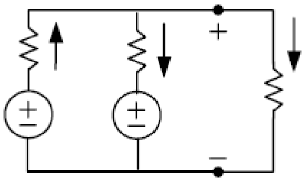
(a) i-v curve of a real current source

(b) Lumped-element model for $|v| \ll R_s i_s$

several sources in connection

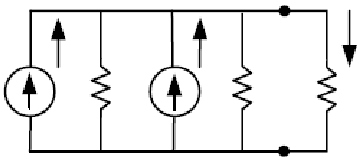
(1) voltage sources: in series to provide a higher output voltage supply

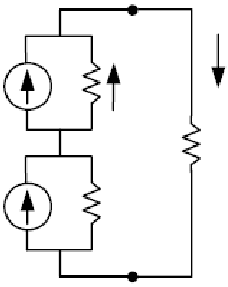




(in parallel)

(2) current sources: in parallel to provide a higher output current supply

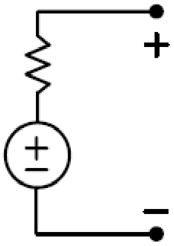




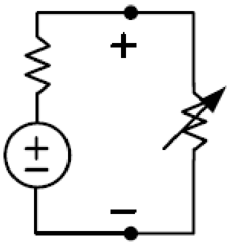
(in series)

measurement of source model

(1) voltage source

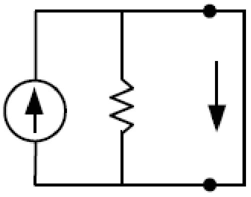


measure o.c. voltage $\rightarrow v_{oc} = v_s$

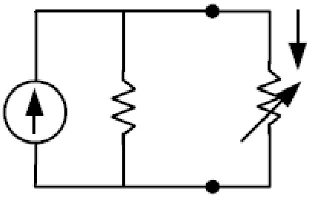


adjust R_L until $v = \frac{v_{oc}}{2} \rightarrow R_L = R_s$

(2) current source

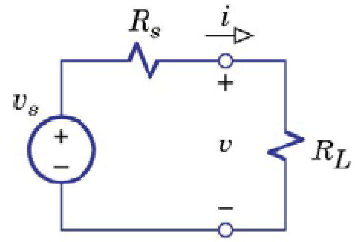


measure s.c. current $\rightarrow i_{sc} = i_s$



adjust R_L until $i = \frac{i_{sc}}{2} \rightarrow R_L = R_s$

3. maximum power transfer: for fixed R_S, the maximum power transferred from source to the load as $R_L=R_S$ and η =50%



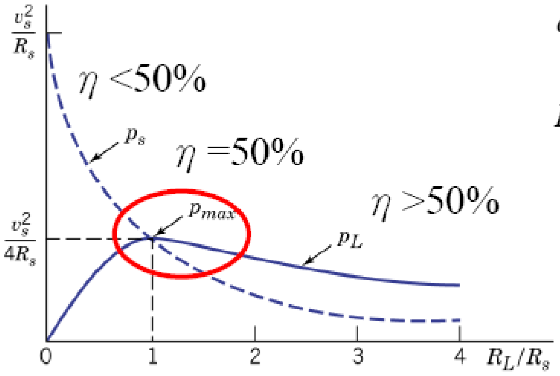
$$p_s = i^2 R_s = \left(\frac{v_s}{R_s + R_L}\right)^2 R_s$$

$$p_L = i^2 R_L = \left(\frac{v_s}{R_s + R_L}\right)^2 R_L$$

$$\frac{dp_L}{dR_L} = 0 \rightarrow \boxed{R_L = R_s}$$

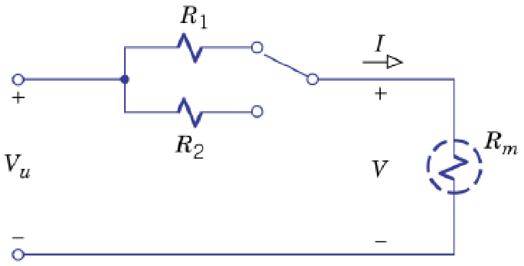
$$p_{\max} = \frac{i_s^2 R_s}{4}, \eta \equiv \frac{p_L}{p_s + p_L}$$

resistance match



Measurement of *electrical quantities* – voltmeter, ammeter and ohmmeter

1. DC voltmeter: a “good” voltmeter should have large R_m

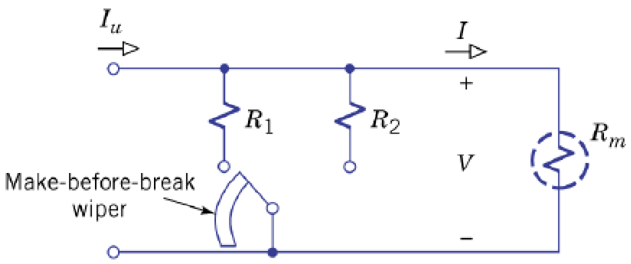


$$V = \frac{R_m}{R_1 + R_m} V_u \rightarrow V_u = \frac{R_1 + R_m}{R_m} V$$

$$\text{full scale } V_{ufs} = \frac{R_1 + R_m}{R_m} V_{fs}$$

$$R_m = 5k\Omega, V_{fs} = 250mV, \begin{cases} R_1 = 15k\Omega \rightarrow V_{ufs} = 1V \\ R_2 = 95k\Omega \rightarrow V_{ufs} = 5V \end{cases}$$

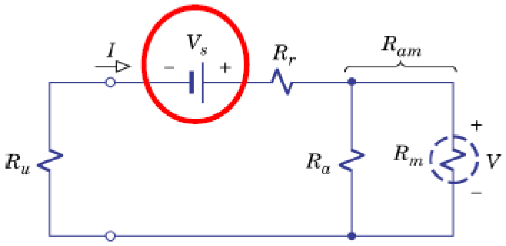
2. DC ammeter: a “good” ammeter should have small R_m



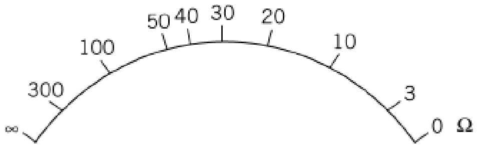
$$I = \frac{R_1}{R_1 + R_m} I_u \rightarrow I_u = \frac{R_1 + R_m}{R_1} I$$

$$\text{full scale } I_{ufs} = \frac{R_1 + R_m}{R_1} I_{fs}$$

3. DC ohmmeter: only measures isolated non-energized resistors and be aware of damaging circuit by internal source V_s



(a) Analog ohmmeter circuit



(b) Nonlinear ohms scale

$$R_u = 0 \rightarrow V_{fs} = \frac{R_{am}}{R_r + R_{am}} V_s, R_{am} = R_a // R_m$$

$$R_u \neq 0 \rightarrow V = \frac{R_{am}}{R_u + R_r + R_{am}} V_s = \frac{R_r + R_{am}}{R_u + R_r + R_{am}} V_{fs} : \text{nonlinear and backwards scale}$$